Lecture 2

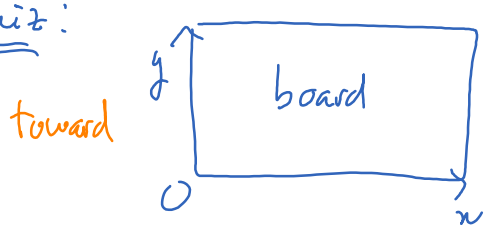
Wednesday, January 13, 2021 2:05 PM

* Prayer
* Spiritual thought: Alma 1:26

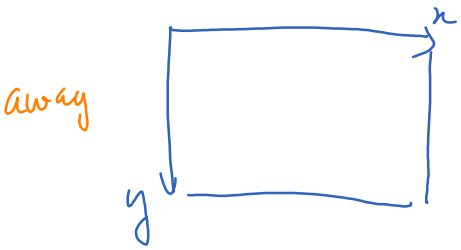
preacher is no better than the hearer,
teacher is no better than the learner. They are equal, and labor according to his strength.

* Answering questions ...

Quiz:



what is the direction of the z-axis?
point toward us or away from us?



About Mathematica: read the instruction file to know how to plot a function using Mathematica.

* Vectors & points

points vs vector

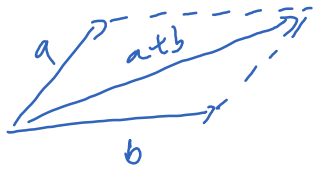
location displacement, velocity, accelaration

$A(1,2,3)$ parathesses

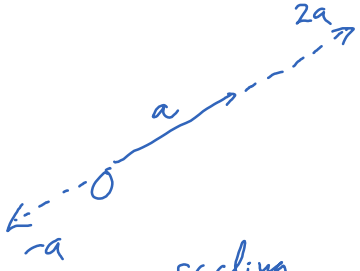
$a = \langle 1, 2, 3 \rangle$ angle brackets

Two style to write a vector:

- Specify the base point & the head point: $\overrightarrow{AB}$
- Only specify the head point. The base point is the origin.

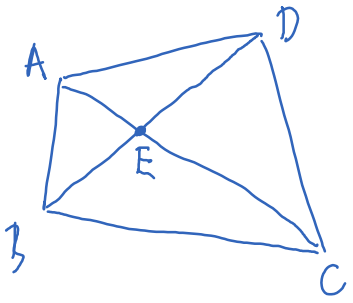


addition



scaling

Ex



$A(1,2,3)$

$B(2,4,2)$

$C(0,1,1)$

$D(2,5,-1)$

Find the coordinates of E?

Although one can do this by finding the equation of the lines AC, BD and their intersection, here we use a slightly different method.

Let $a = \overrightarrow{EA}$

$b = \overrightarrow{ED}$

Because A, C, E are on the same line, $a = k\overrightarrow{AC}$

Because B, D, E are on the same line, $b = l\overrightarrow{BD}$.

We have

$$a + \overrightarrow{AD} = \overrightarrow{EA} + \overrightarrow{AD} = \overrightarrow{ED} = b .$$

Thus,

$$k\overrightarrow{AC} + \overrightarrow{AD} = l\overrightarrow{BD}$$

Note that $\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = \langle -1, -1, -2 \rangle$

$\overrightarrow{AD} = \langle 1, 3, -4 \rangle$

$\overrightarrow{BD} = \langle 0, 1, -3 \rangle$

Then

$$k\langle -1, -1, -2 \rangle + \langle 1, 3, -4 \rangle = l\langle 0, 1, -3 \rangle$$

$$\begin{cases} -k + 1 = 0 \\ -k + 3 = l \\ -2k - 4 = -3l \end{cases} \rightsquigarrow \begin{cases} k = 1 \\ l = 2 \end{cases}$$

Then $\overrightarrow{EA} = a = \overrightarrow{AC} = \langle -1, -1, -2 \rangle$.

$$\overrightarrow{OE} = \overrightarrow{OA} + \overrightarrow{AE} = \overrightarrow{OA} - \overrightarrow{EA} = \langle 1, 2, 3 \rangle - \langle -1, -1, -2 \rangle$$
$$= \langle 2, 3, 5 \rangle$$

Dot product:

$$a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3$$

Length:

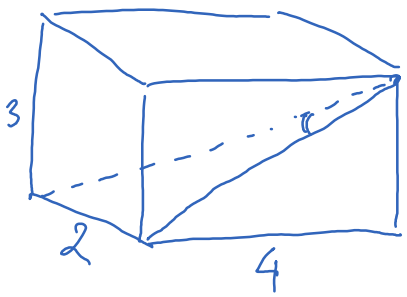
$$|a| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

Angle:

$$\cos\theta = \frac{a \cdot b}{|a||b|}$$

(Coming from the Law of Cosine of Triangle)

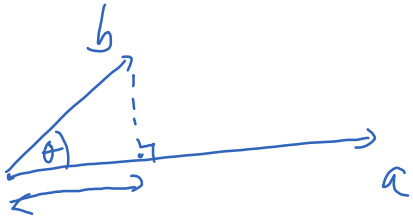
Ex



What is angle θ?

Direction angles, cosines.

Projection:



scalar projection:

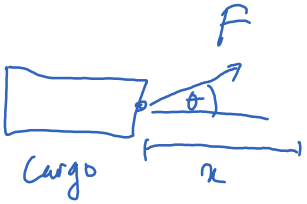
$$\text{comp}_b a = |b| \cos\theta$$

$$= |b| \frac{a \cdot b}{|a||b|} = \frac{a \cdot b}{|a|}$$

vector projection:

$$\text{proj}_b a = \frac{a}{|a|} \quad \text{comp}_a a = \frac{a \cdot b}{|a|^2} a$$

Ex:



$$\text{work} = \vec{F} \cdot \vec{x} = F x \cos\theta$$

maximum when $\theta = 0$.

Direction angles & direction cosines